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Nessie Notation: A New Tool in Sequential Substitution Systems and Graph Theory for Summarizing Concatenations

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Abstract

While doing research looking for ways to categorize causal networks generated by Sequential Substitution Systems, I created a new notation to compactly summarize concatenations of integers or strings of integers, including infinite sequences of these, in the same way that sums, products, and unions of sets can be summarized. Using my method, any sequence of integers or strings of integers with a closed-form iterative pattern can be compactly summarized in just one line of mathematical notation, including graphs generated by Sequential Substitution Systems, many Primitive Pythagorean Triplets, and various Lucas sequences including the Fibonacci sequence and the sequence of square triangular numbers.

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I. INTRODUCTION

A Sequential Substitution System (SSS) is akin to a mathematical yeast mold that uses simple rules to grow and develop in complex ways [1]. By tracking when and where each rule is applied, a kind of graph known as a causal network can be derived from an SSS, describing the behavior of its growth mathematically with an infinite sequence of numbers connected by arrows. An infinite sequence of integer strings can then be derived from the casual network by taking the differences between numbers in the network, giving us what is known as the Reduced Net Differences (RND) sequence for that SSS in ongoing research at Southern Adventist University. In previous research, the most compactly that the behavior of an SSS could be summarized was in the infinite RND sequence form, but while attempting to find a reliable way to determine the dimensionality of a given SSS, I developed a method for compactly summarizing sequences of both integers and strings of integers, including infinite sequences of these, a method that I call Net Summary (Nessie) Notation. My notation is similar to sum, product, and union notations, but instead of concatenating each sequential element with a plus, multiplier, or union, respectively, Nessie Notation concatenates each sequential element with a comma, thus creating a sequence built from iterative components. Using this method, I was able to find repeating patterns in the RND sequence of each SSS that I studied and thus compressed the entire infinite sequences down to just one line of mathematical notation each. I then tested the process in reverse and was able to reconstruct the infinite RND sequence and then the infinite causal network from the Nessie Notation. I also discovered that the number of dimensions in which the causal network grows is always one greater than the highest power present on n within the Nessie Notation, except in the case of exponential growth, in which case n is the power on some integer. I quickly realized that my new notation had broader application than only SSS's, and condensed the Fibonacci sequence, the sequence of Lucas numbers, the Pell sequence, and the Jacobsthal sequence to one line each using a Binet-like formula for each in conjunction with my Nessie Notation; similarly, I also condensed the sequence of numbers that are both triangular and perfectly square using an analogous formula. Finally, I summarized a sequence that I derived of all Primitive Pythagorean Triplets $\{a, b, c\}$ such that hypotenuse c is the largest possible number for a given smallest leg a, enumerated through all positive integers a, using Nessie Notation, demonstrating that my notation has the potential to do things beyond what had been possible with set-builder notation.

II. METHODS

Take the SSS in Fig. 1. By tracking when and where each rule is used, I generate the

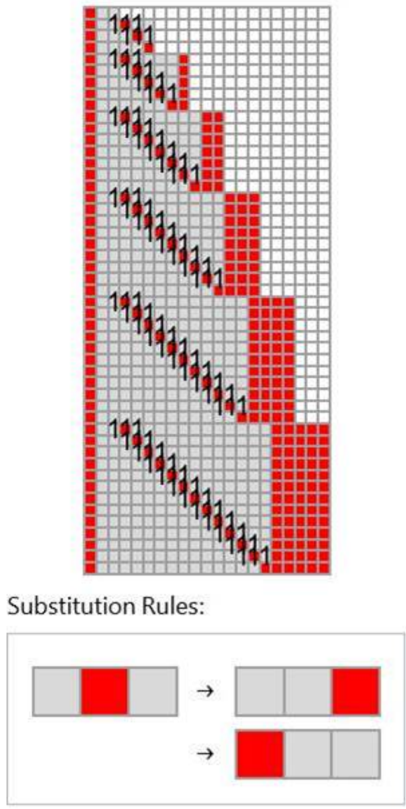

FIG. 1. A Sequential Substitution System

causal network in Fig. 2, which can be expressed mathematically as follows:

$\{1 \to 2, 1 \to 5, 1 \to 5, 2 \to 3, 2 \to 3, 2 \to 6, 3 \to 7, 3 \to 8, 4 \to 5, 4 \to 10, 4 \to 10, 5 \to 6, 5 \to 6, 5 \to 11, 6 \to 7, 6 \to 7, 6 \to 12, 7 \to 8, 7 \to 8, 7 \to 13, 8 \to 14, 8 \to 15, 9 \to 10, 9 \to 17, 9 \to 17, 10 \to 11, 10 \to 11, 10 \to 18, 11 \to 12, 11 \to 12, 11 \to 19, 12 \to 13, 12 \to 13, 12 \to 20, 13 \to 14, 13 \to 14, 13 \to 21, 14 \to 15, 14 \to 15, 14 \to 22, 15 \to 23, 15 \to 24, 16 \to 17, 16 \to 26, 16 \to 26, 17 \to 18, 17 \to 18, 17 \to 27, 18 \to 19, 18 \to 19, 18 \to 28, 19 \to 20, 19 \to 20, 19 \to 29, 20 \to 21, 20 \to 21, 20 \to 30, 21 \to 22, 21 \to 22, 21 \to 31, 22 \to 23, 22 \to 23, 22 \to 32, 23 \to 24, 23 \to 24, 23 \to 33, 24 \to 34, 24 \to 35, 25 \to 26, 25 \to 37, 25 \to 37, 26 \to 27, 26 \to 27, 26 \to 38, 27 \to 28, 27 \to 28, 27 \to 39, 28 \to 29, 28 \to 29, 28 \to 40, 29 \to 30, 29 \to 30, 29 \to 41, 30 \to 31, 30 \to 31, 30 \to 42, 31 \to 32, 31 \to$

$32,31 \to 43,32 \to 33,32 \to 33,32 \to 44,33 \to 34,33 \to 34,33 \to 45,34 \to 35,34 \to 35,34 \to 46,35 \to 47,35 \to 48,36 \to 37,37 \to 38,37 \to 38,38 \to 39,38 \to 39,39 \to 40,39 \to 40,40 \to 41,40 \to 41,41 \to 42,41 \to 42,42 \to 43,42 \to 43,43 \to 44,43 \to 44,44 \to 45,44 \to 45,45 \to 46,45 \to 46,46 \to 47,46 \to 47,47 \to 48,47 \to 48,...\}$.

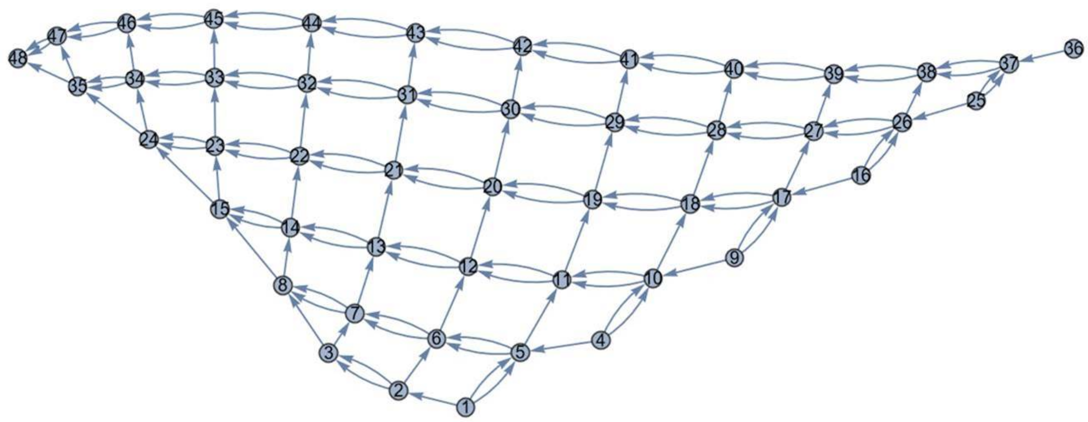

FIG. 2. A Causal Network

I then take the RND sequence of this network, giving the following infinite sequence of integer strings:

$\{\{1,4,4\}, 1\cdot\{1,1,4\}, \{4,5\}, \{1,6,6\}, 3\cdot\{1,1,6\}, \{6,7\}, \{1,8,8\}, 5\cdot\{1,1,8\}, \{8,9\}, \{1,10,10\}, 7\cdot\{1,1,10\}, \{10,11\}, \{1,12,12\}, 9\cdot\{1,1,12\}, \{12,13\}, ...\}$.

I now break the RND sequence into chunks of length 3:

$\{\{1,4,4\}, 1\cdot\{1,1,4\}, \{4,5\}$,

$\{1,6,6\}$,$3\cdot\{1,1,6\}, \{6,7\}$,

$\{1,8,8\}$,$5\cdot\{1,1,8\}, \{8,9\}$,

$\{1,10,10\}$,$7\cdot\{1,1,10\}, \{10,11\}$,

$\{1,12,12\}$,$9\cdot\{1,1,12\}, \{12,13\}, ...\}$.

Expressing each integer a in each chunk as a function $f_a(n)$ of the chunk number n yields the following arbitrary chunk:

$\{1, 2(n+1), 2(n+1)\}, (2(n-1)+1)\cdot\{1,1,2(n+1)\}, \{2(n+1), 2(n-1)+5\}$.

As there are no discarded strings at the beginning, I let the iteration begin at $n=1$ and, as this is an infinite RND sequence, continue for an infinite number of iterations, finally giving the Nessie Notation summary of the RND:

$$\underset{n=1}{\overset{\infty}{\mathbb{S}}} [\{1, 2(n+1), 2(n+1)\}, (2(n-1)+1)\cdot\{1,1,2(n+1)\}, \{2(n+1), 2(n-1)+5\}].$$

I now rederive the RND from this notation as follows:

$\{\{1,4,4\}, 1\cdot\{1,1,4\}, \{4,5\}, \{1,6,6\}, 3\cdot\{1,1,6\}, \{6,7\}, \{1,8,8\}, 5\cdot\{1,1,8\}, \{8,9\}, \{1,10,10\}, 7\cdot\{1,1,10\}, \{10,11\}, \{1,12,12\}, 9\cdot\{1,1,12\}, \{12,13\}, ...\}$,

and from here the causal network of the SSS:

$\{1 \to 2, 1 \to 5, 1 \to 5, 2 \to 3, 2 \to 3, 2 \to 6, 3 \to 7, 3 \to 8, 4 \to 5, 4 \to 10, 4 \to 10, 5 \to 6, 5 \to 6, 5 \to 11, 6 \to 7, 6 \to 7, 6 \to 12, 7 \to 8, 7 \to 8, 7 \to 13, 8 \to 14, 8 \to 15, 9 \to 10, 9 \to 17, 9 \to 17, 10 \to 11, 10 \to 11, 10 \to 18, 11 \to 12, 11 \to 12, 11 \to 19, 12 \to 13, 12 \to 13, 12 \to 20, 13 \to 14, 13 \to 14, 13 \to 21, 14 \to 15, 14 \to 15, 14 \to 22, 15 \to 23, 15 \to 24, 16 \to 17, 16 \to 26, 16 \to 26, 17 \to 18, 17 \to 18, 17 \to 27, 18 \to 19, 18 \to 19, 18 \to 28, 19 \to 20, 19 \to 20, 19 \to 29, 20 \to 21, 20 \to 21, 20 \to 30, 21 \to 22, 21 \to 22, 21 \to 31, 22 \to 23, 22 \to 23, 22 \to 32, 23 \to 24, 23 \to 24, 23 \to 33, 24 \to 34, 24 \to 35, 25 \to 26, 25 \to 37, 25 \to 37, 26 \to 27, 26 \to 27, 26 \to 38, 27 \to 28, 27 \to 28, 27 \to 39, 28 \to 29, 28 \to 29, 28 \to 40, 29 \to 30, 29 \to 30, 29 \to 41, 30 \to 31, 30 \to 31, 30 \to 42, 31 \to 32, 31 \to 32, 31 \to 43, 32 \to 33, 32 \to 33, 32 \to 44, 33 \to 34, 33 \to 34, 33 \to 45, 34 \to 35, 34 \to 35, 34 \to 46, 35 \to 47, 35 \to 48, 36 \to 37, 37 \to 38, 37 \to 38, 38 \to 39, 38 \to 39, 39 \to 40, 39 \to 40, 40 \to 41, 40 \to 41, 41 \to 42, 41 \to 42, 42 \to 43, 42 \to 43, 43 \to 44, 43 \to 44, 44 \to 45, 44 \to 45, 45 \to 46, 45 \to 46, 46 \to 47, 46 \to 47, 47 \to 48, 47 \to 48, ...\}$,

thus demonstrating that the entire behavior of the SSS can be summarized in just one line of mathematical notation.

III. RESULTS

One problem in analyzing an SSS and its causal network is to determine the number of dimensions in which the network is growing. Ongoing research at Southern Adventist University has led to the discovery of the four clues, algorithms that analyze a causal network and return an estimate as to the number of dimensions in which it is growing. The fourth clue is currently believed to be 100% correct in its estimates but has the added caveat of not always returning an estimate at all. In the Nessie Notation summary of a causal network, I have found that the highest power polynomial of variable n present in the notation always corresponds to one less that the estimate given by the fourth clue, except when the fourth clue determines the network to be exponential in dimension, in which case n is a power on some integer within the Nessie Notation. (I have not yet found a case where the form n^n

has occurred within my notation, but I hypothesize that it would correspond to an entirely new dimension of causal network that has yet to be encountered from an SSS.) In addition to causal networks generated by SSS's, the third and fourth clues can be applied to many other types of graphs as well, and analysis has revealed that using Nessie Notation on these graphs displays the same rule as above.

For example, taking the Nessie Notation summary from the example above, the highest power on n is 1, thus predicting that the causal network is growing in $1+1=2$ dimensions. Taking the fourth clue for this network results in the determination that the network is indeed growing in 2 dimensions.

IV. DISCUSSION

Nessie Notation can also be applied to many sequences of integers that have closed-form generating formulae, such as the Fibonacci sequence [2]:

$$\mathop{\mathbb{S}}_{n=0}^{\infty}\left[\frac{(1+\sqrt{5})^n-(1-\sqrt{5})^n}{2^n\sqrt{5}}\right]=\{0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,1597,...\},$$

the sequence of Lucas numbers [3]:

$$\mathop{\mathbb{S}}_{n=0}^{\infty}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n+\left(\frac{1-\sqrt{5}}{2}\right)^n\right]=\{2,1,3,4,7,11,18,29,47,76,123,199,322,521,843,1364,...\},$$

the Pell sequence [4]:

$$\mathop{\mathbb{S}}_{n=0}^{\infty}\left[\frac{(1+\sqrt{2})^n-(1-\sqrt{2})^n}{2\sqrt{2}}\right]=\{0,1,2,5,12,29,70,169,408,985,2378,5741,13860,33461,...\},$$

and the Jacobsthal sequence [5]:

$$\mathop{\mathbb{S}}_{n=0}^{\infty}\left[\frac{2^n-(-1)^n}{3}\right]=\{0,1,1,3,5,11,21,43,85,171,341,683,1365,2731,5461,...\},$$

as well as the sequence of numbers that are both triangular and perfectly square [6]:

$$\mathop{\mathbb{S}}_{n=0}^{\infty}\left[\left(\frac{(3+2\sqrt{2})^n-(3-2\sqrt{2})^n}{4\sqrt{2}}\right)^2\right]=\{0,1,36,1225,41616,1413721,48024900,1631432881,$$

$55420693056,\ 1882672131025,\ 63955431761796,\ ...\}$.

The sequence of all Primitive Pythagorean Triplets of the form $\{a,b,c\}$ such that hypotenuse c is the largest possible number for a given smallest leg a, enumerated through each positive integer a that can be the smallest leg of a PPT, can be summarized by Nessie Notation as follows:

$$\{3,4,5\},\{5,12,13\},\mathop{\mathbb{S}}_{n=1}^{\infty}[\{4n+3,8n^2+12n+4,8n^2+12n+5\},$$

$$\{4n+4,4n^2+8n+3,4n^2+8n+5\},\{4n+5,8n^2+20n+12,8n^2+20n+13\}]=$$

$$\{3,4,5\},\{5,12,13\},\{7,24,25\},\{8,15,17\},\{9,40,41\},\{11,60,61\},\{12,35,37\},...,$$

thus demonstrating that this new notation has the capacity to do things that were previously impossible with only set-builder notation, even outside the realm of SSS's.

V. CONCLUSION

Just like sum, product, and union notations before it, Nessie Notation has the power to unlock new areas of mathematics that were previously unreachable. This reiterates the timeless truth that the world of mathematics is more complex and beautiful than anyone can imagine today, but that with the innovations and ideas of tomorrow, we someday might see it all.

[1] E. W. Weisstein and T. Rowland, "Sequential substitution system," (2022).

[2] E. W. Weisstein, "Binet's fibonacci number formula," (2022).

[3] E. W. Weisstein, "Lucas number," (2022).

[4] E. W. Weisstein, "Pell number," (2022).

[5] E. W. Weisstein, "Jacobsthal number," (2022).

[6] E. W. Weisstein, "Square triangular number," (2022).